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Contract: Electromagnetic Inverse Scattering
N00014-89-K0003

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The contract involves the 3–dimensional inverse electromagnetic problem for lossy and gradient type material. In particular it is concerned with developing methods for determining the material properties of a scatterer from reflection data.

The approach that has been taken has been to generalize the method of wave splitting and invariant imbedding (which has been so successful in the one–dimensional case) from the one–dimensional case to three dimensions. Because of the complexity of the problems this was applied first to the wave equation, then the dissipative wave equation.

The accomplishments are as follows:

(i) The procedure of planar wave splitting in an non–homogeneous medium has been successfully generalized to apply the 3–dimensional case for the dissipative (and even more general cases) wave equation. See [3], [5].

(ii) The imbedding equation and various properties of the reflection kernel that are needed in the direct and inverse problems, have been obtained for the general (non–stratified) 3–dimensional wave equation [4].

(iii) An outline of an algorithm for the direct and inverse problems has been developed for the 3–dimensional wave equation using the invariant imbedding equations for the reflection operator. Some analysis (such as removal of singularities) still remains to be done to make it practical [4].

(iv) Numerical implementation of the inverse problem associated with the dissipative wave equation for a stratified medium has been carried out. The algorithm was based upon taking transverse moment of the imbedding equation for the reflection operator. Both the velocity and the dissipation factor can be obtained from one–sided data only, namely using measurements on a surface, of the scattered field produced by a point impulsive source [6].

Importance of Research

Apart from the particular results quoted above, the main importance of the research carried out under this contract, is that it has provided a basis for future development of a practical 3–dimensional inverse algorithm for using transient data produced by point impulsive sources. In particular, since the termination of the contract, the wave splitting technique has been successfully developed for Maxwell's equation with dispersive terms. Recently a practical algorithm has been developed for the 3–dimensional wave equation, where rather than using the imbedding equations for the reflection operator as given in [4], the equations associated with the wave–splitting of the field produced by a point impulsive source were used.

The technique developed under this contract have been taken up by other researchers, particularly in Sweden.

Presentations

[1] "Wave Splitting and the Reflection Operator for the Wave Equation," R.C.P. 264 meeting at Montpellier France (November 27–December 1, 1989).

[2] "The Reflection Operator for the Wave Equation in $\mathbf{R}^3$" Conference "Methoden und Verfahren der Mathematischen Physik," Oberwolfach (November 26–December 2, 1989).

[3] "Wave Splitting for a Class of Hyperbolic Equations," S.I.A.M. Symposium on "Invariant Imbedding and the Inverse Problems," Albuquerque, NM (April 19–20, 1990).

Papers

[4] "Invariant Imbedding for the Wave Equation in Three Dimensions and the Applications to the Direct and Inverse Problems," Journal Inverse Problems, **6**, p. 1075, (1990).

[5] "Root of a Second Order Hyperbolic Differential Operator and Wave–Splitting," to appear in special S.I.A.M. proceedings Oct. (1992).

[6] "Inverse Problem for the Dissipative Wave Equation in a Stratified Half–space and Linearization of the Imbedding Equations, with S. He, Journal Inverse Problems, **8**, p. 435 (1992).